Positron beam loading in uniform regime

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What do we need for plasma based positron acceleration?

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- **Focusing force**

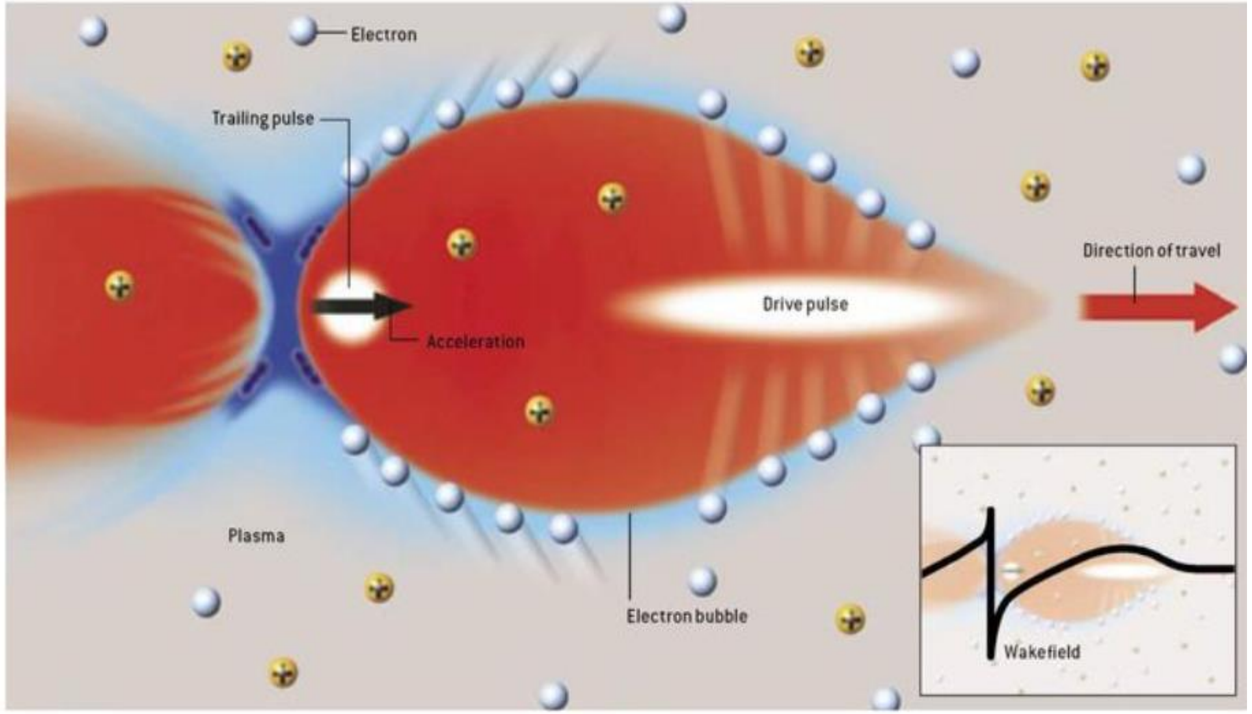
  $-\int_0^{r_0}[\rho_e(r') - J_{ze}(r')/c]r'\,dr' > \frac{1}{2}r_0^2$ in a uniform ion background. (A region where the plasma electrons exceed the ion density)

- **Uniform Ez along the beam**

  E_z can be flattened by the loaded positron beam

- **The ability to simultaneously achieve high efficiency and high quality acceleration**

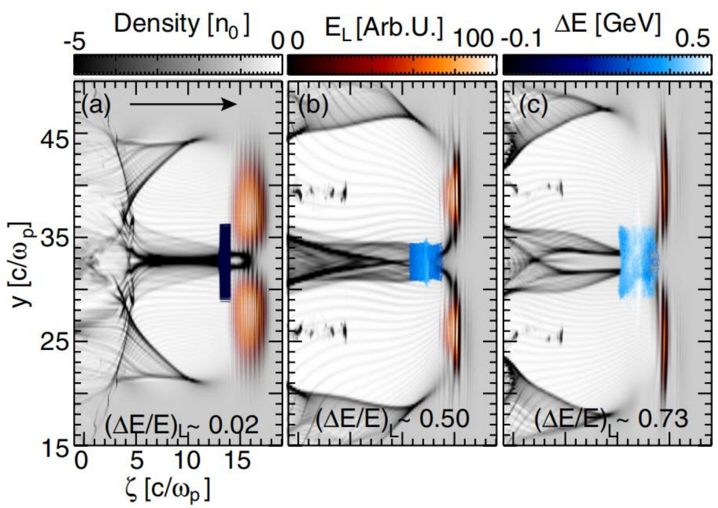
Electron acceleration in the blowout regime

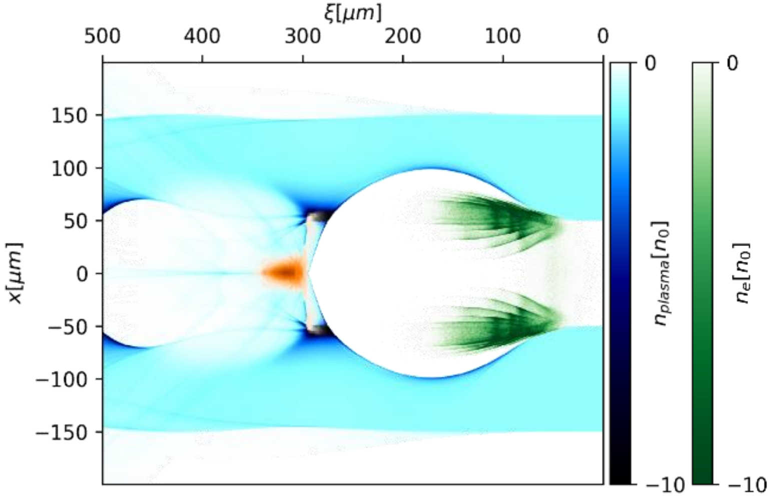


How to focus a positron beam in plasma?

In previous research, we focus the positron beam by tailoring the drive bunch or plasma.

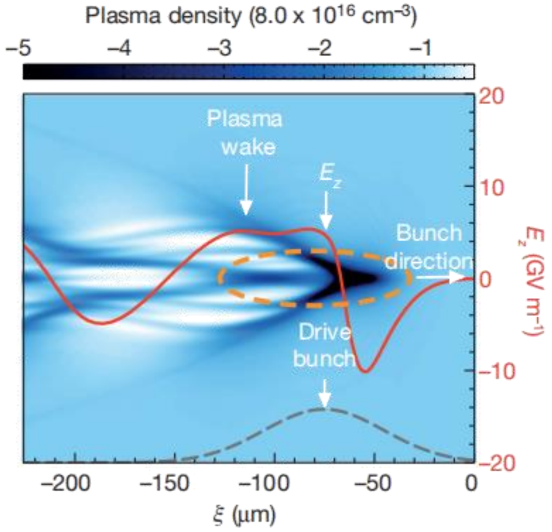
Hollow electron / laser driver

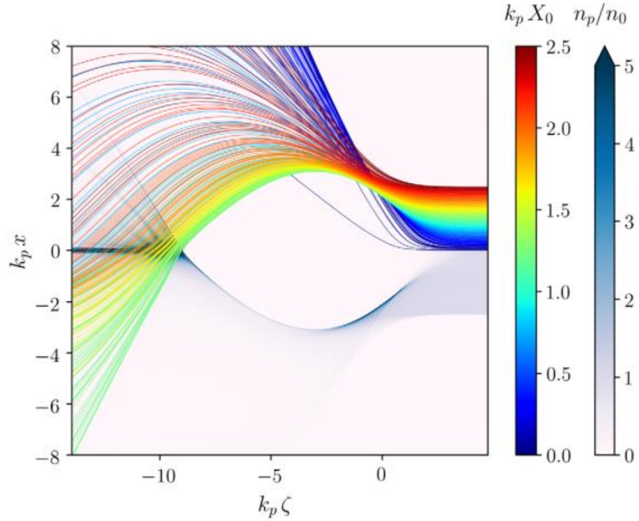




Hollow plasma channel

Positron drive beam





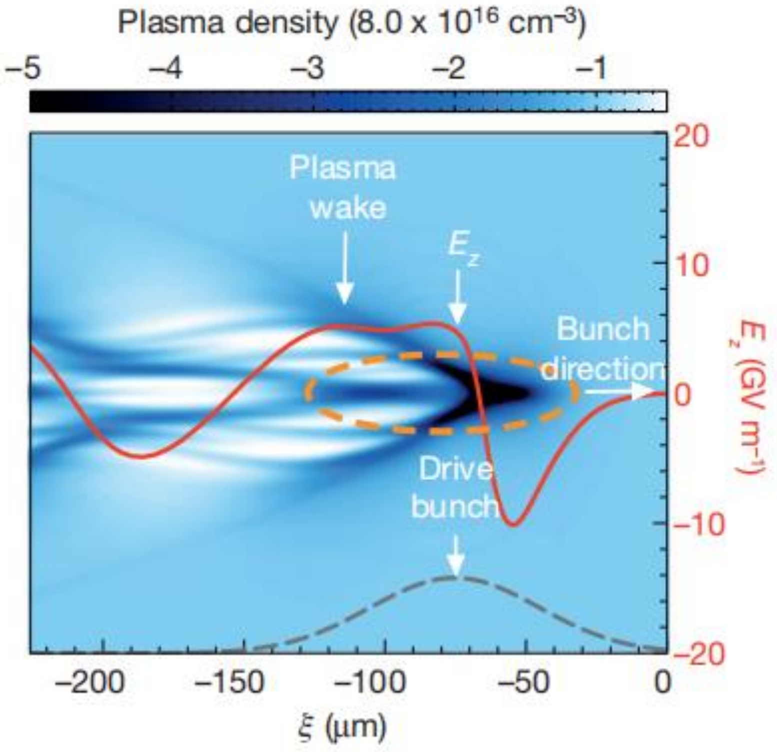
Finite width plasma

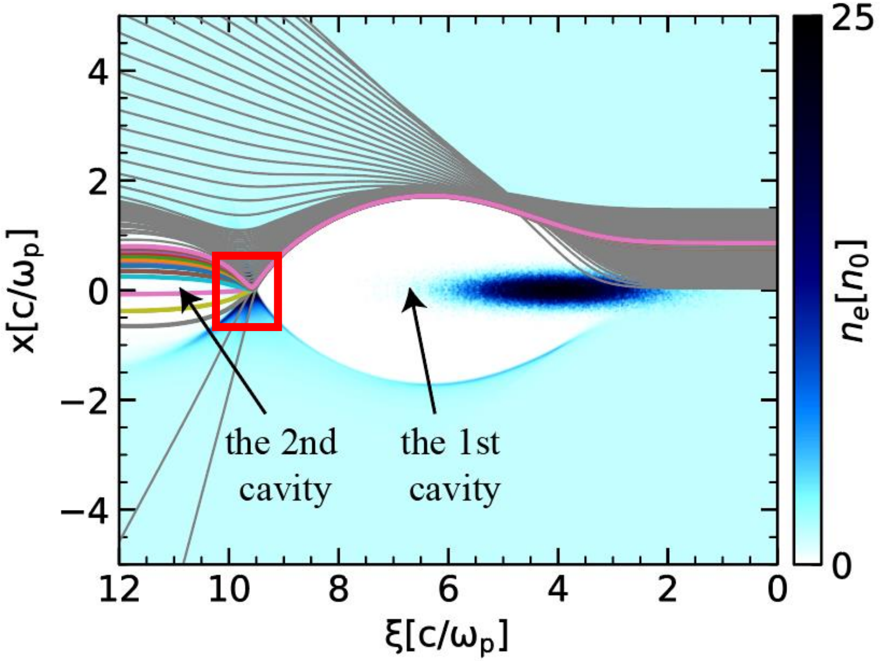
J. Vieira *et al.*, PRL 112, 215001 (2014)
N. Jain *et al.*, PRL 115, 195001 (2015)
S. Diederichs *et al.*, PRAB 22, 081301 (2019)
S. Zhou *et al.*, PRL 127, 174801 (2021)
S. Corde *et al.*, Nature 524, 442 (2015)

Focus positron beam in the blowout regime

- The criterion for focusing positron beam in PWFA

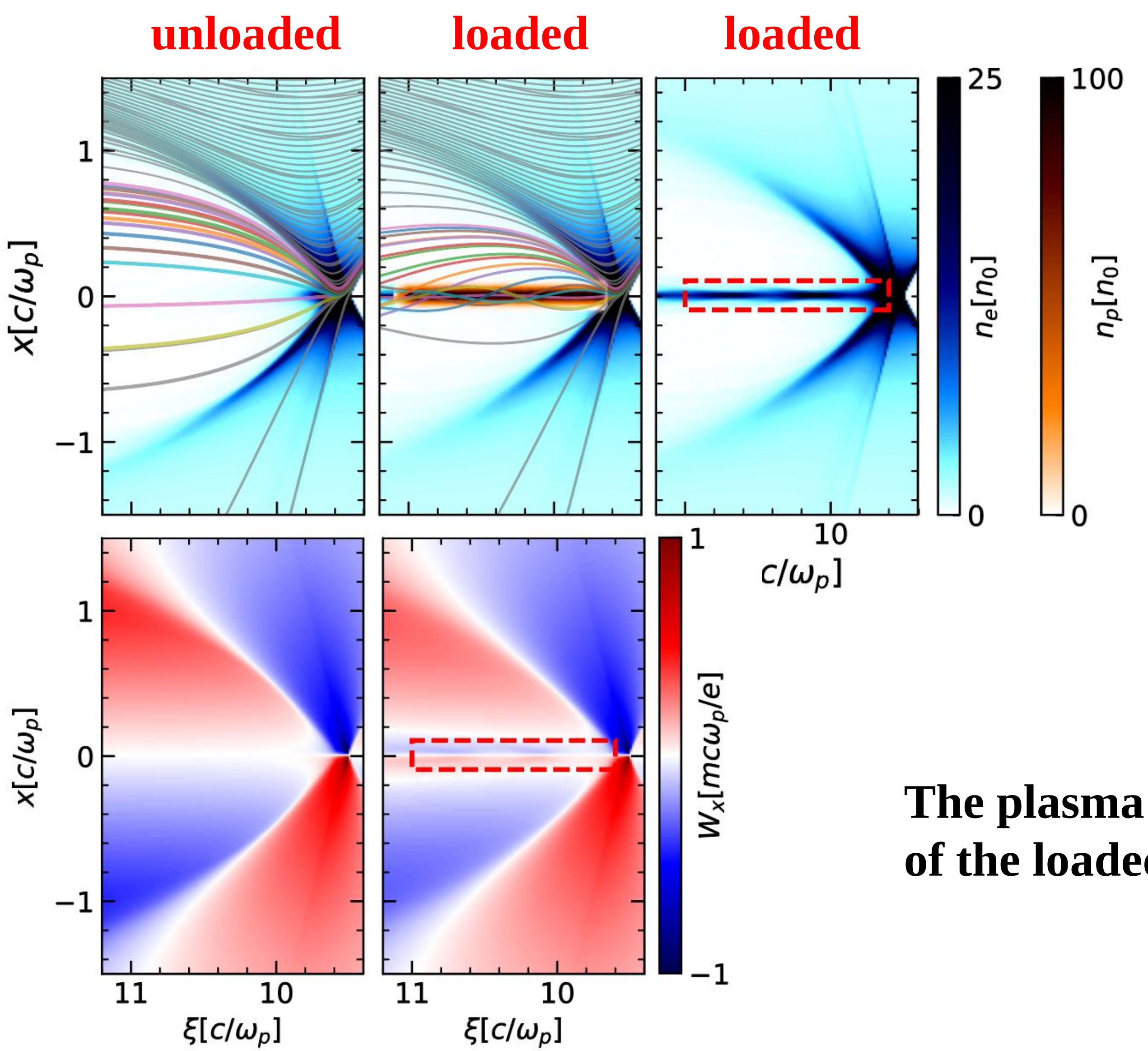
  - We need enough on-axis plasma electrons $-\int_0^{r_0}[\rho_e(r') - J_{ze}(r')/c]r'\,dr' > \frac{1}{2}r_0^2$





We can achieve the same goal in the blowout regime by positron beam loading.

Intense positron beam load in the blowout regime



$$F_{\perp e^-}(r) = -\frac{r}{2} - \frac{1-v_z}{r}\int_0^r n_b r' dr' - \frac{1}{r}\left[\int_0^r \rho_e r' dr' - v_z \int_0^r \frac{J_{ze}}{c} r' dr'\right] - \frac{1-v_z}{2}\int_0^r \frac{\partial E_z}{\partial \xi} dr',$$

Drive beam: $n_{e^-}/n_0 = 23$, $k_p\sigma_z = 1$, $k_p\sigma_r = 0.17$

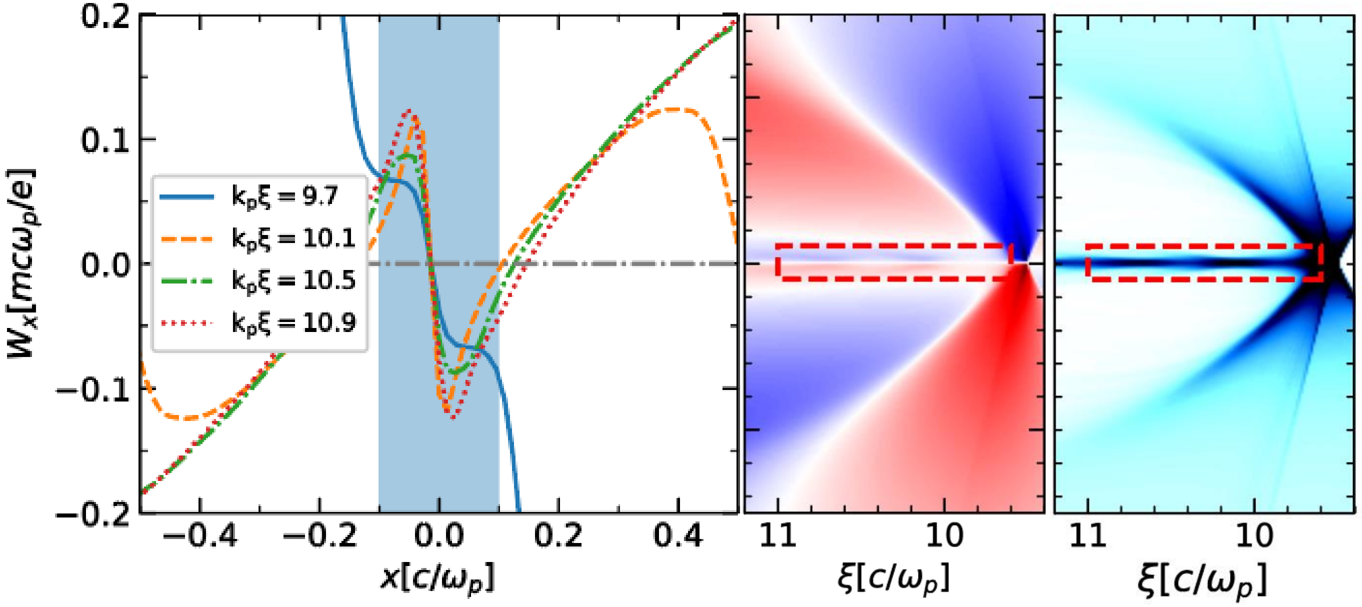
Trailing beam: $n_{e^+}/n_0 = 100$, $k_p l = 1.4$, $\mathrm{k_p\sigma_r} = 0.033$

For $n_p = 3.11 \times 10^{16}\mathrm{cm}^{-3}$, $Q_d = 1.4\mathrm{nC}$, $Q_t = 130\mathrm{pC}$

The plasma electrons are bended by the strong focusing fields of the loaded positron beam and form an on-axis filament.

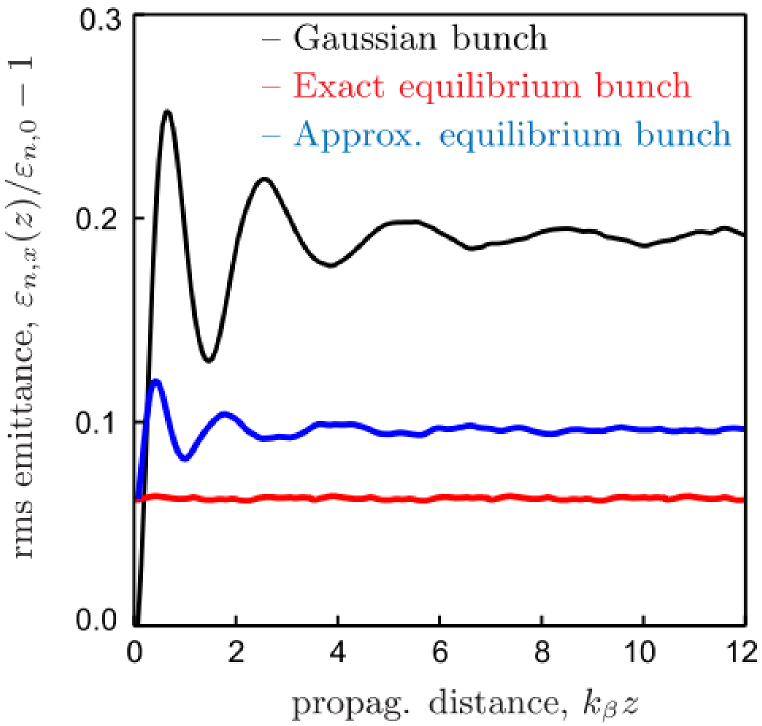
Structure of the focusing force

- Due to the intense interaction between the plasma electron and the loaded positron beam, the distribution of electrons are highly nonuniform, which leads to the focusing field nonlinear in *r* and variable in *z* direction.



- In fact, even with the nonlinear focusing strength the emittance growth can be well controlled.

e^- beam propagation in plasma with ion motion
$n_{b,0}/n_0 = 12000$

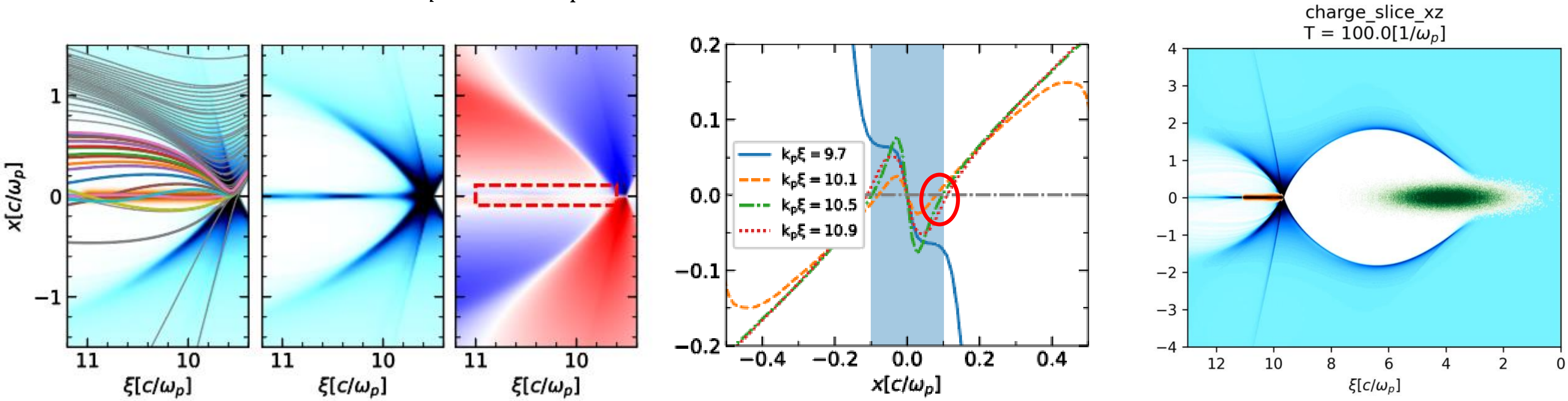


PRAB 20, 111301 (2017)
PRAB 23, 121301 (2020)

Structure of the focusing force

An intense and narrow positron beam is necessary to generate the focusing force for the entire beam in a highly blowout regime, or it will disperse during the long distance propagation.

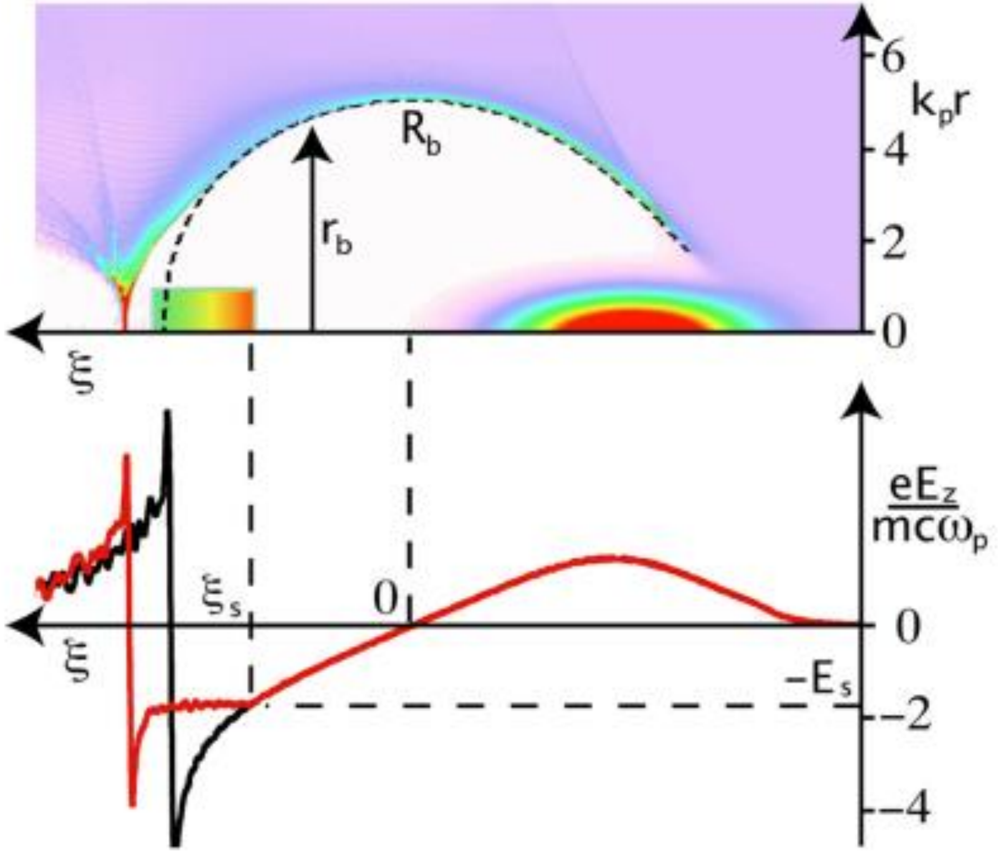
Trailing beam: $n_{e^+}/n_0 = 50$, $k_p l = 1.4$, $\mathrm{k_p \sigma_r} = 0.033$

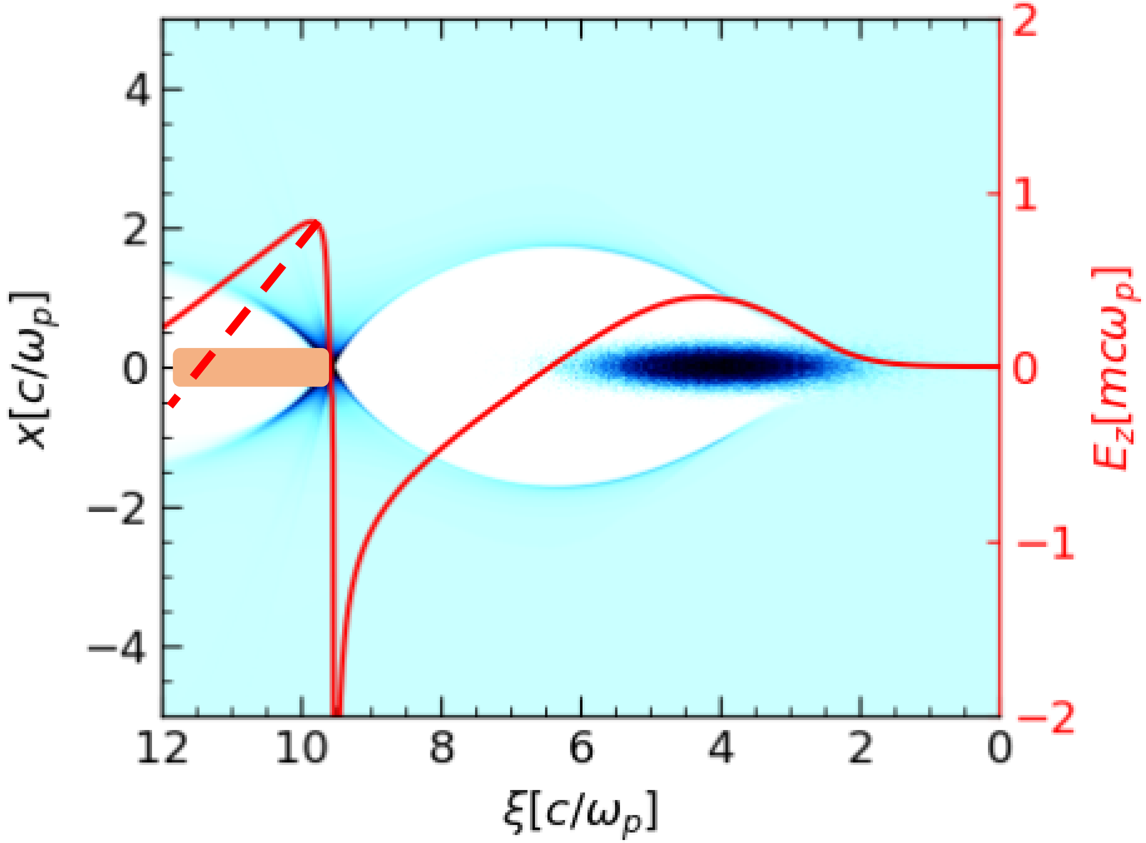


The positron beam loading process also depends on the structure of the bubble, so the requirements for the positron beam parameters are different for situation with a less intense drive beam.

Positron beam loading

Intuitively, beam loading absorbs energy from the wakefields and lowers E_z such that positron beam loading in the blowout regime is impossible to flatten the E_z along the beam.





Positron beam loading effects – original blowout theory

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- Start from the *Lu et al*. theory of the blowout regime.

$$\psi(r,\xi) = \psi_0(\xi) - \frac{r^2}{4} = \frac{r_b^2(\xi)}{4}(1+\beta) - \frac{r^2}{4}$$

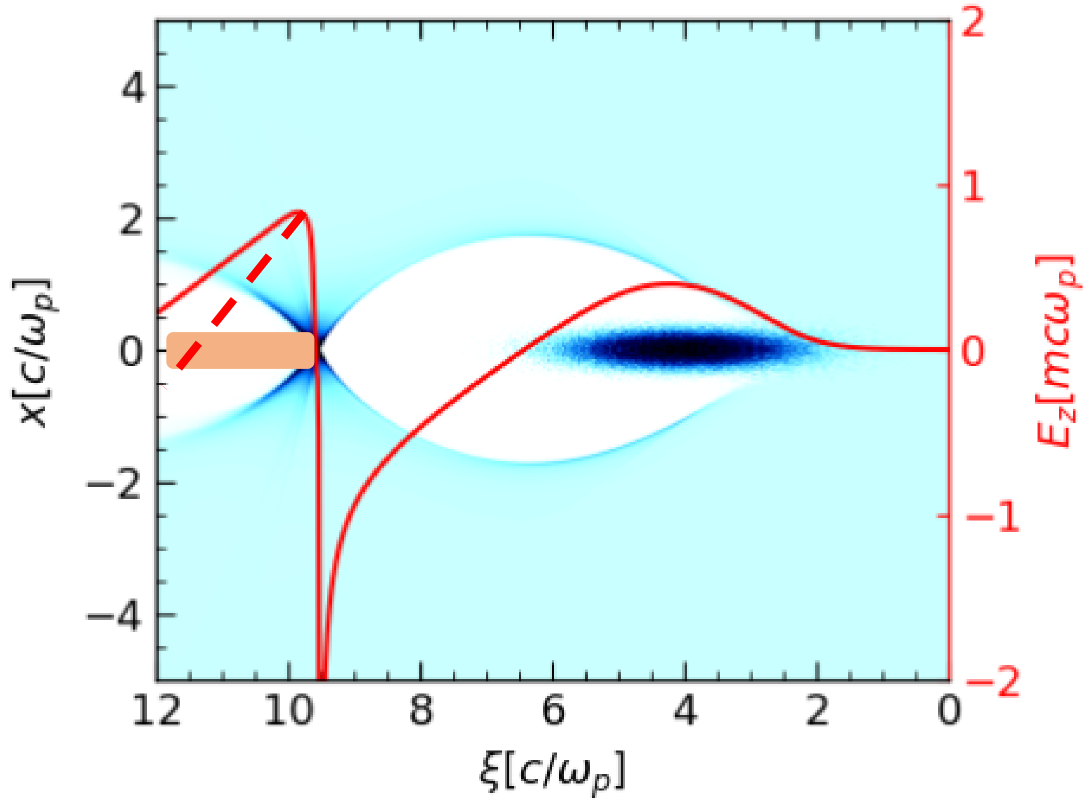
$$E_z(\xi) \approx \frac{1}{2} r_b \frac{dr_b}{d\xi}$$

- At the front of the 2nd bucket, due to the space charge force of the ions inside the cavity

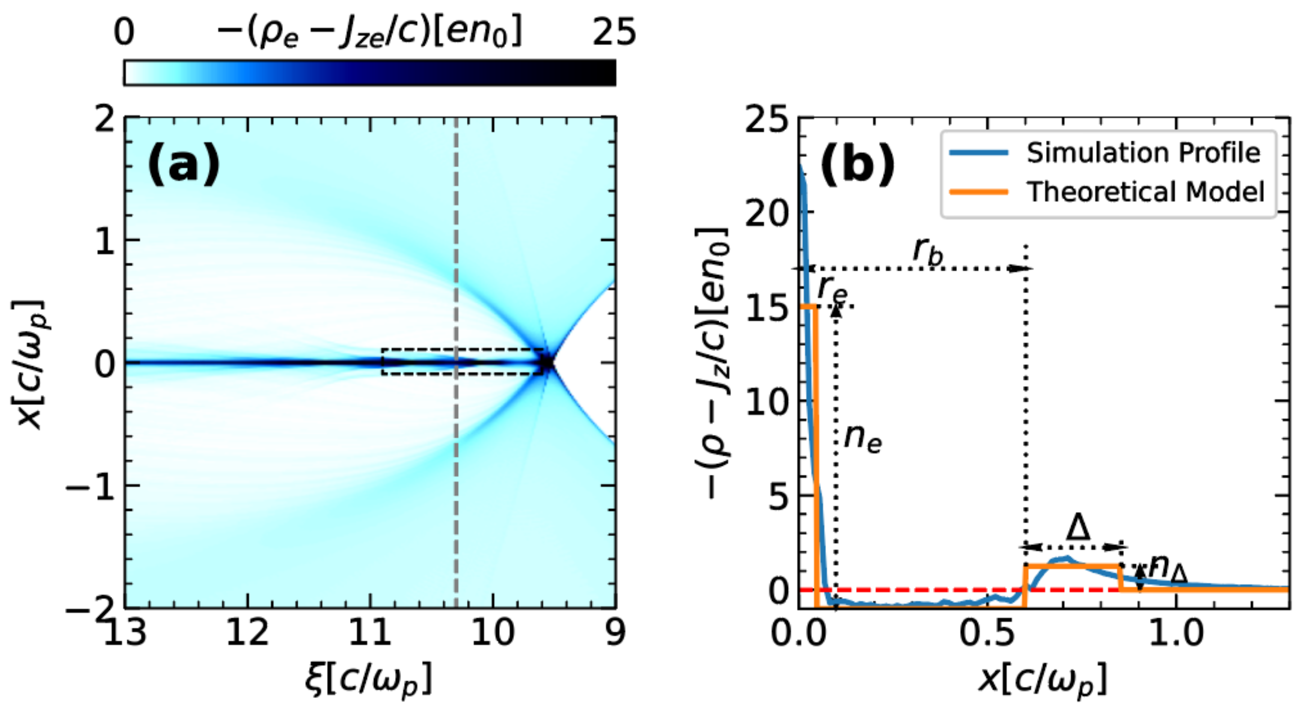
$$\frac{dr_b}{d\xi} \downarrow \text{ and } r_b \uparrow$$

- And $E_z(\xi)$ decreases for the unloaded case.
- If the positron beam is loaded at the front of the 2nd bucket, $\frac{dr_b}{d\xi}$ decreases more quickly and r_b increases more slowly, $\frac{1}{2} r_b \frac{dr_b}{d\xi}$ or $E_z(\xi)$ will decrease with a larger slope.

- We obtain the same conclusion that the positron beam cannot get uniform acceleration in the blowout regime.



Positron beam loading effects – modified theory



Actually, the on-axis electron filament must be included.

- Define $\nabla_\perp^2 \psi = -(\rho - J_z/c) \equiv S$
- The continuity of charge and current requires

$$(n_e - 1) r_e^2 - \left(r_b^2 - r_e^2\right) + n_\Delta\left[(r_b + \Delta)^2 - r_b^2\right] = 0$$

- We can obtain $\psi_0 \equiv \psi(r = 0)$

$$\psi_0(\xi) = \frac{r_b^2(\xi)}{4}\big(1 + \beta(\xi)\big) - \frac{1}{4} n_e(\xi) r_e^2(\xi)\big(1 + \beta(\xi)\big) - \frac{1}{2} n_e(\xi) r_e^2(\xi) \ln\frac{r_b(\xi)}{r_e(\xi)}$$

and $E_{z0} \equiv E_z(r = 0)$

- Assume a uniform electron filament along ξ

$$\left(\frac{\partial r_e}{\partial \xi} = \frac{\partial n_e}{\partial \xi} = 0\right)$$

$$E_{z0}(\xi) = \frac{\partial \psi_0(\xi)}{\partial \xi} = \frac{\partial}{\partial \xi}\left[\frac{r_b^2(\xi)}{4}\big(1 + \beta(\xi)\big)\right] + \left[-\frac{1}{2} n_e r_e^2 \frac{1}{r_b} \frac{\partial r_b(\xi)}{\partial \xi}\right]$$

Positron beam loading effects – modified theory

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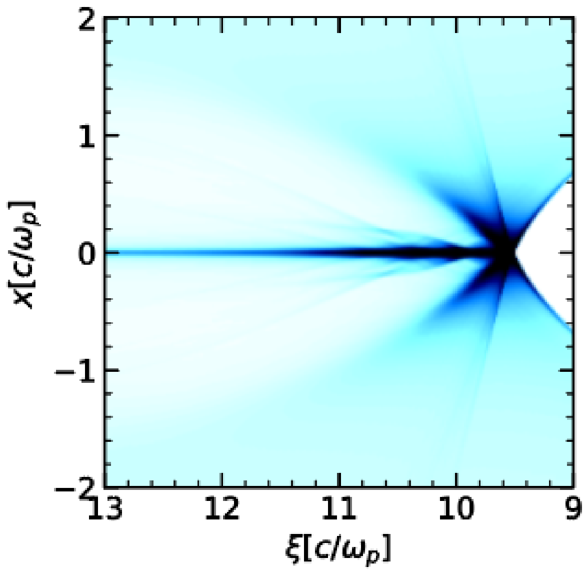
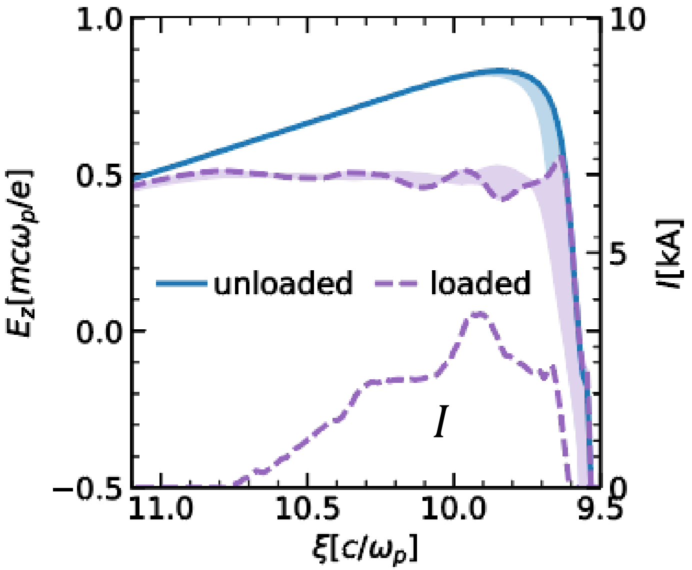
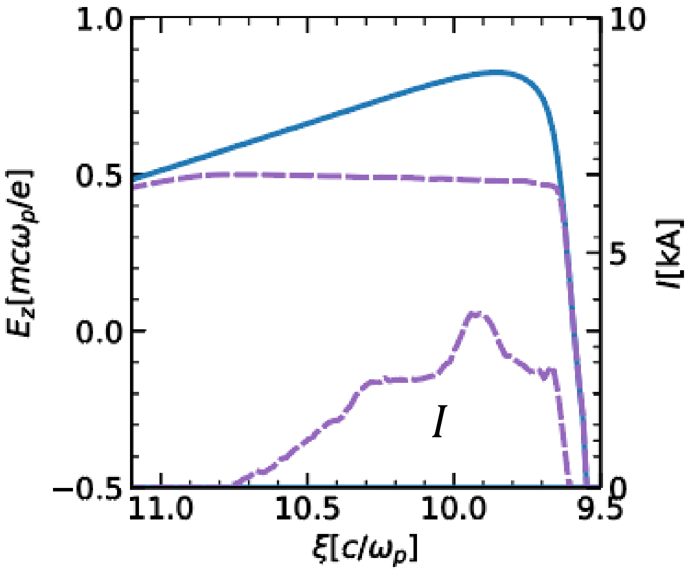
$$E_{z0}(\xi)=\frac{\partial}{\partial\xi}\left[\frac{r_b^2(\xi)}{4}\left(1+\beta(\xi)\right)\right]+\left[-\frac{1}{2}n_e r_e^2\frac{1}{r_b}\frac{\partial r_b(\xi)}{\partial\xi}\right]\approx\frac{1}{2}r_b\frac{dr_b}{d\xi}+\left[-\frac{1}{2}n_e r_e^2\frac{1}{r_b}\frac{\partial r_b(\xi)}{\partial\xi}\right]\equiv E_{z1}+E_{z2}$$

- As ξ increases, E_{z1} decreases in amplitude as in the previous case. At the same time, since $r_b\uparrow$ and $\frac{dr_b}{d\xi}\downarrow$, E_{z2} also decreases in amplitude but has the opposite sign to E_{z1}. It leads to the possibility that $E_{z0}(\xi)$ can be flattened.





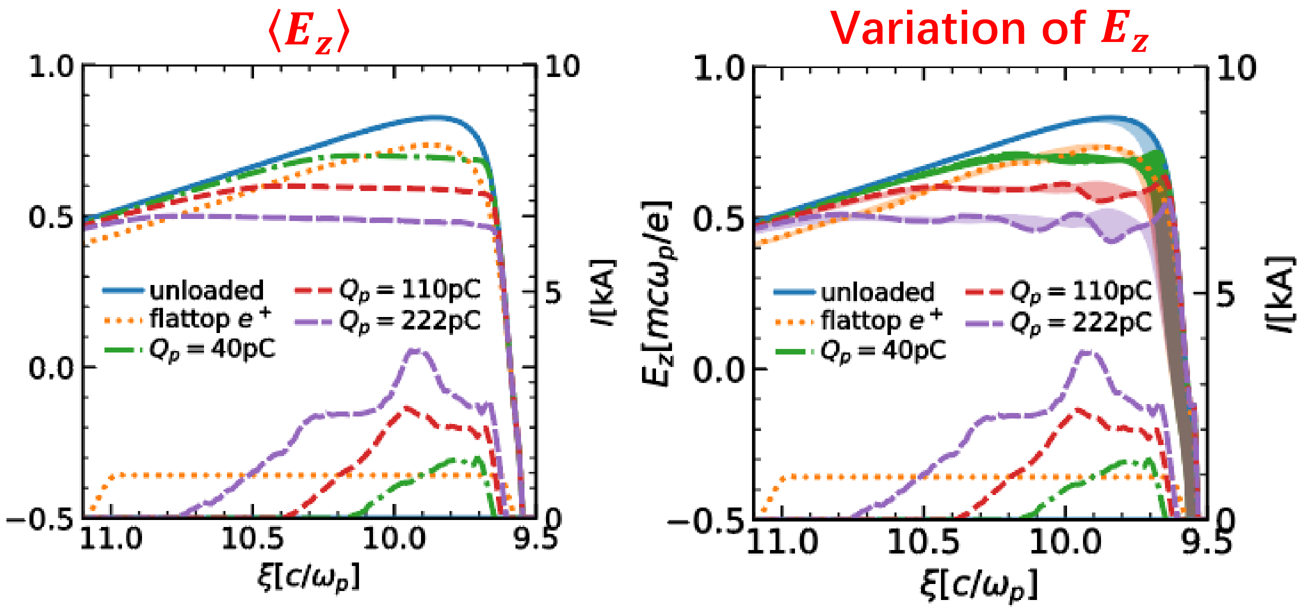
$\langle E_z\rangle$

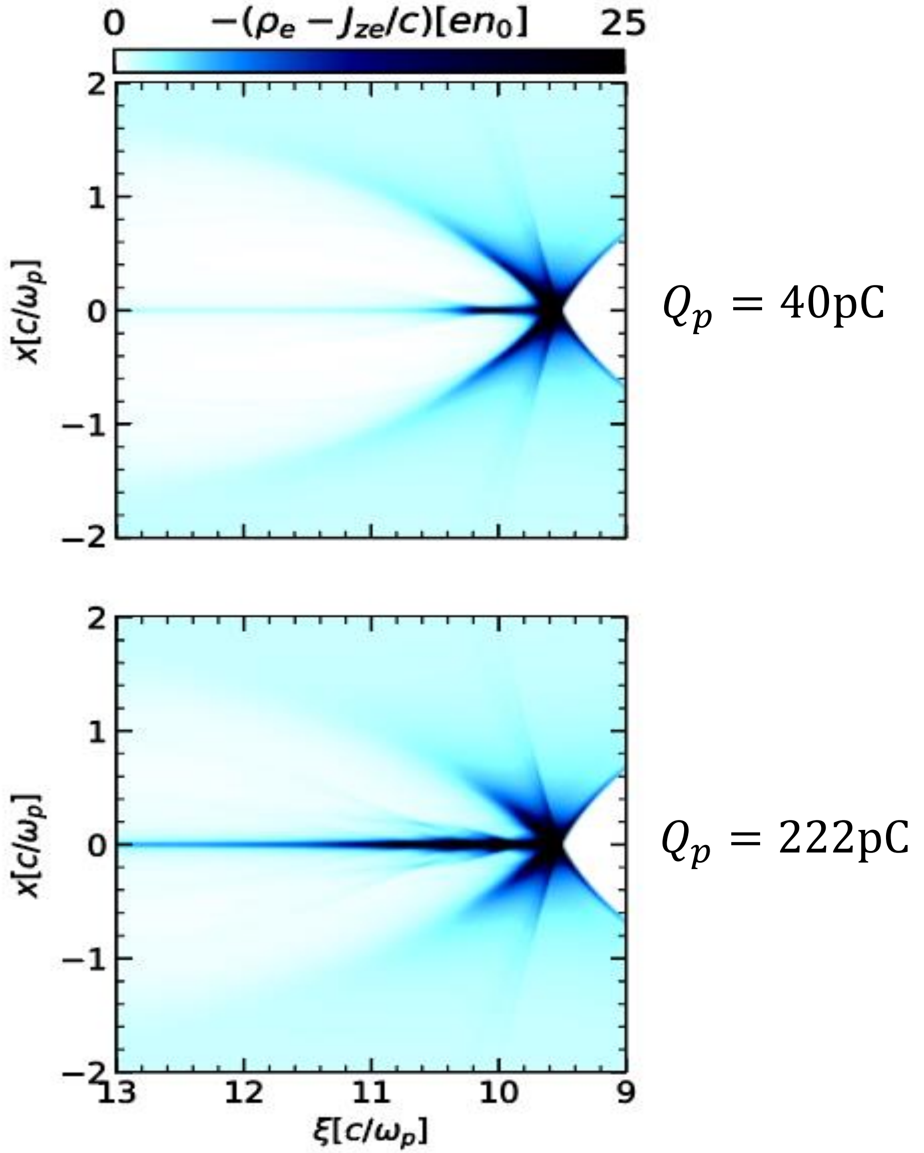


Positron beam loading effects

According to **the Panofsky-Wenzel theorem:**

$$\frac{\partial E_z(r,\xi)}{\partial r} = \frac{\partial W_\perp(r,\xi)}{\partial \xi} = \frac{\partial \Psi(r,\xi)}{\partial r \partial \xi} = \frac{\partial}{\partial \xi}\left(-\frac{1}{r}\int_0^r [\rho_e(r',\xi) - J_{ze}(r',\xi)/c] r' \, dr'\right)$$





Positron beam with higher charge can lead to higher energy transfer efficiency, but the stronger oscillation of the plasma electrons will increase the slice energy spread for the positron beam.

Positron beam acceleration of long distance

Drive beam:

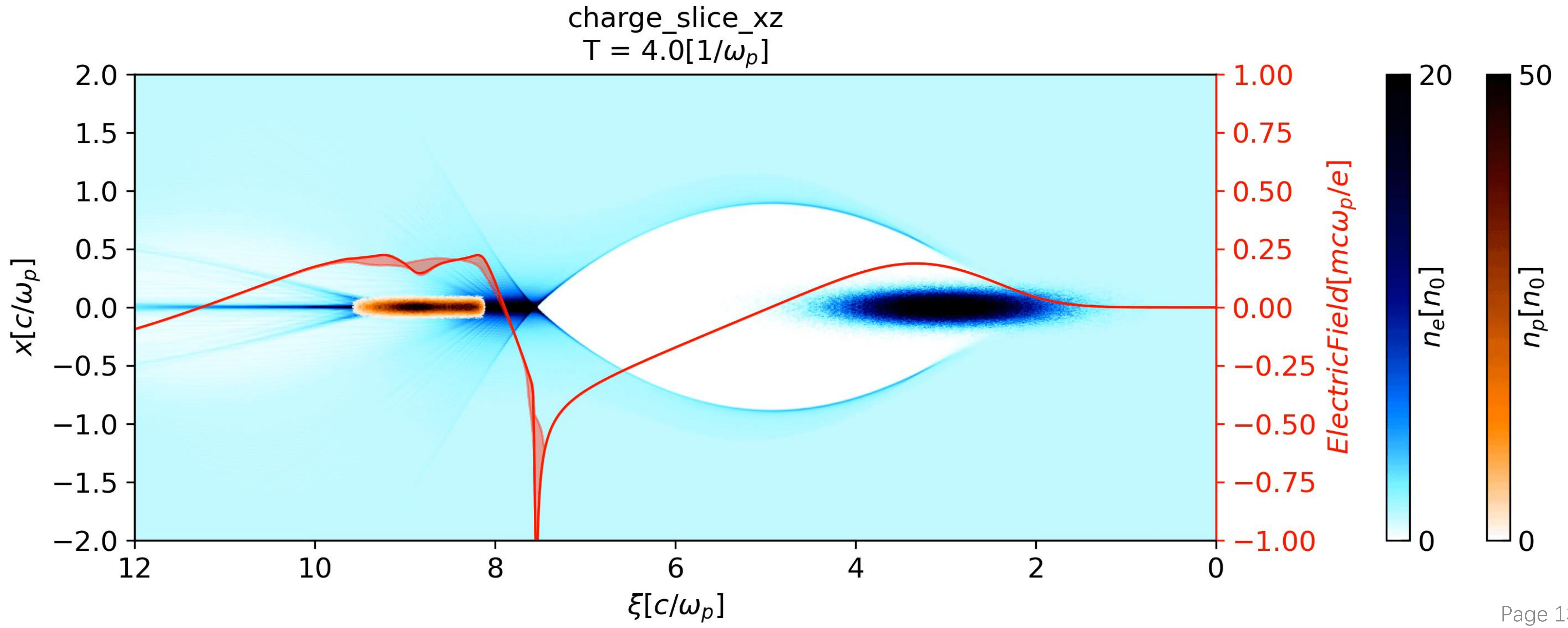
$Q_d = 534\text{pC}, \sigma_r = 5\mu\text{m}, \sigma_z = 40\mu\text{m}, E_0 = 2.5\text{GeV}$

$\epsilon_n = 0.5\text{mm} \cdot \text{mrad}\ (k_p\xi < 2)$ and $20.8\text{mm} \cdot \text{mrad}\ (k_p\xi \geq 2)$

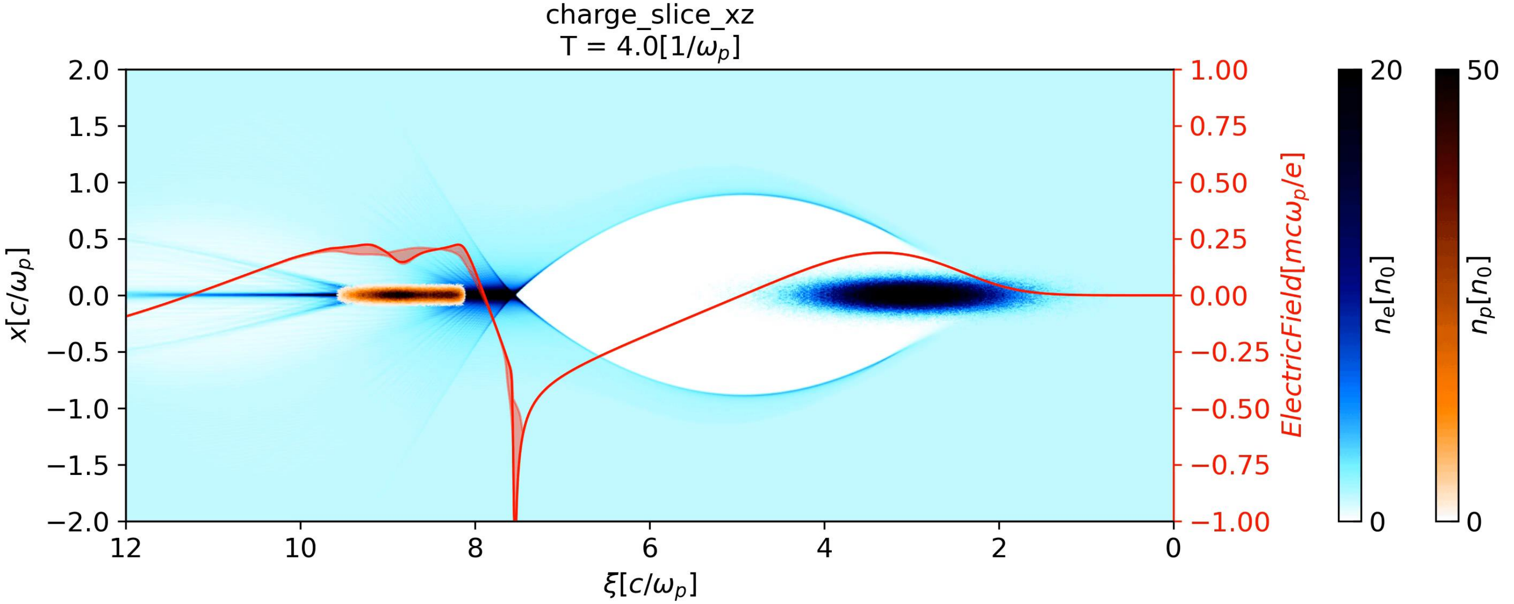
Trailing beam:

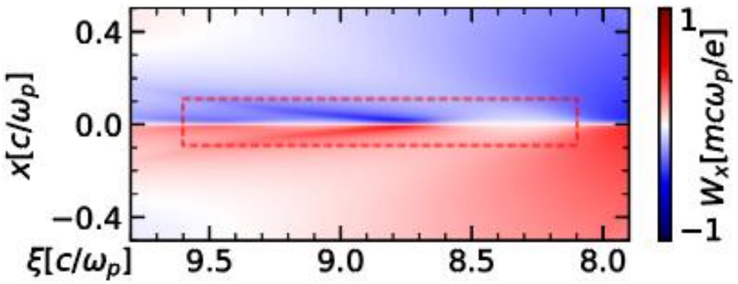
$Q_t = 98\text{pC}, \sigma_r = 2\mu\text{m}, \epsilon_n = 6\text{mm} \cdot \text{mrad}, E_0 = 2.5\text{GeV}$

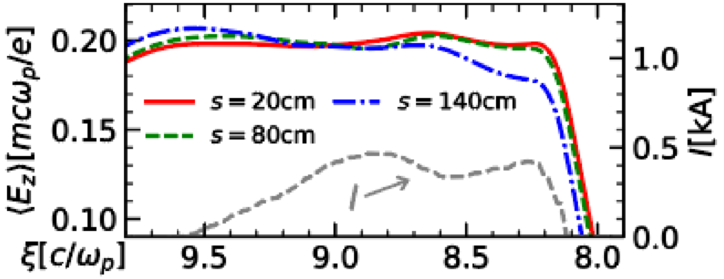
Plasma: $n_p = 7.8 \times 10^{15}\text{cm}^{-3}$

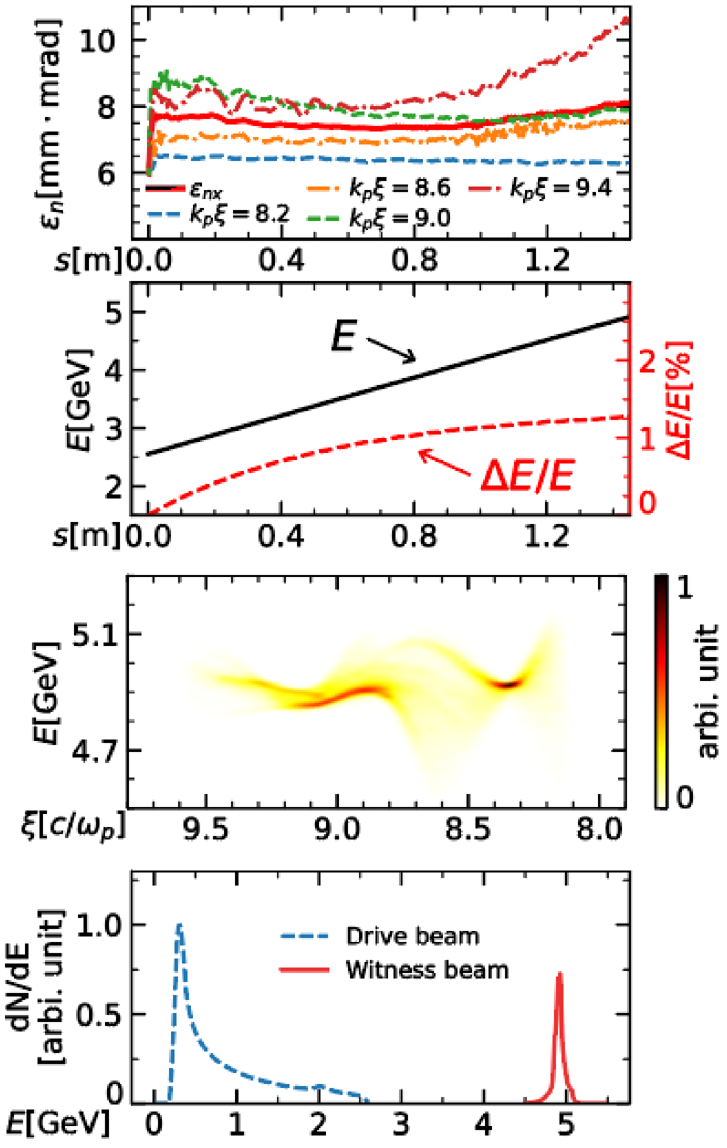


Positron beam acceleration of long distance









$\Delta_{rms} = 1.27\%, \quad \eta = 25.5\%, \quad \bar{E}_z = 1.64\mathrm{GV/m}$

Electron-positron acceleration in the blowout regime

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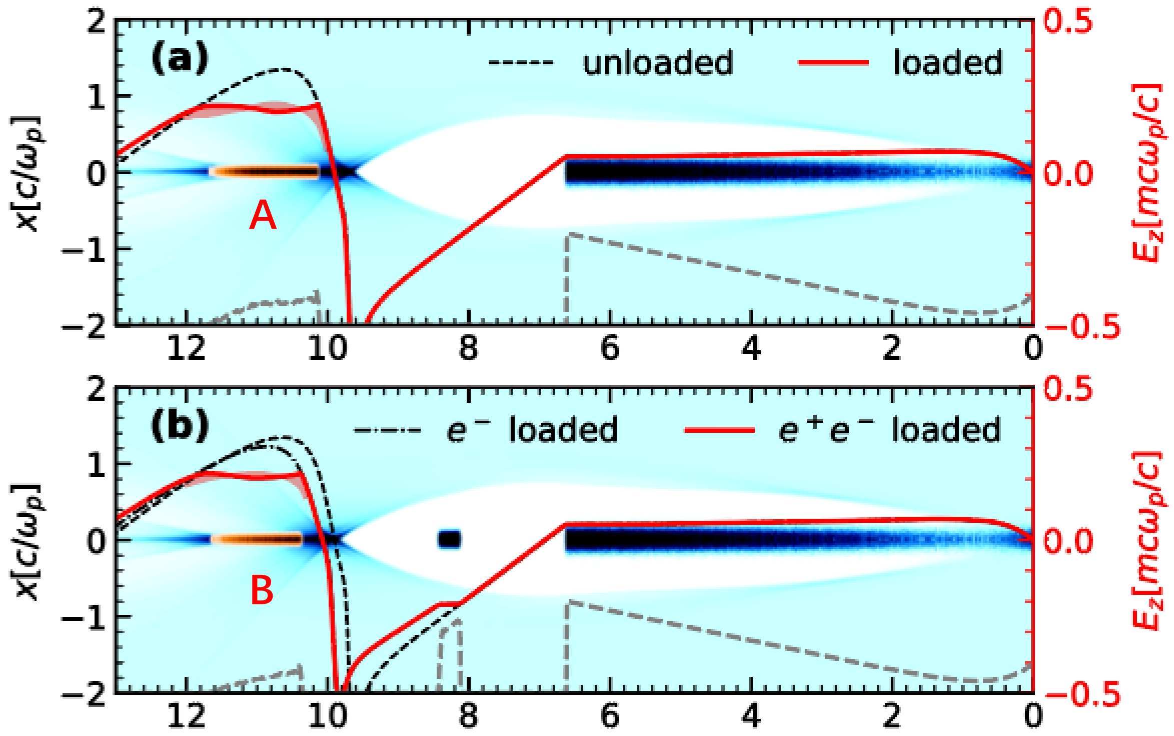
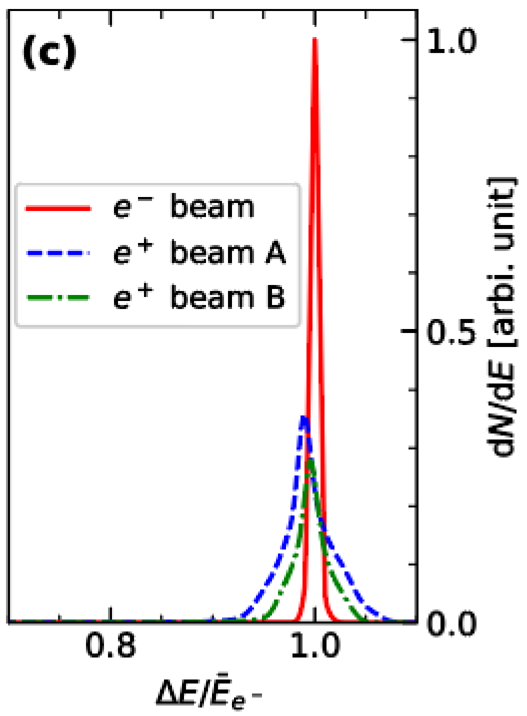
- Loading an extra electron beam in the same wake can improve the energy spread for positron beam and the overall beam loading efficiency.

witness beam	Q[pC]	$q\bar{E}$[GeV/m]	Δ_{rms}[%]	η[%]
e^+ beam A	131	1.78	2.75	35.3
e^+ beam B	80.4	1.78	2.03	21.3
e^- beam	84.0	1.78	0.29	22.8





Shiyu Zhou *et al.*, arXiv.2211.07962

A brief summary

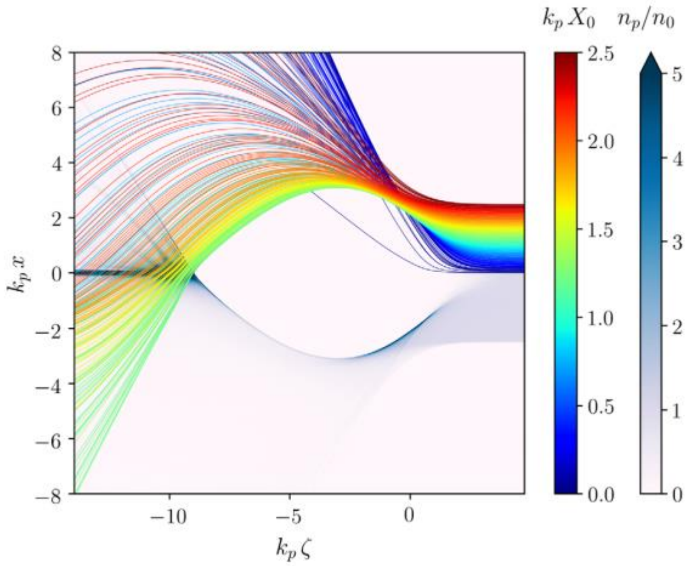
- Focusing force ☺
  An intense positron beam can absorb enough plasma electrons to form an on-axis filament

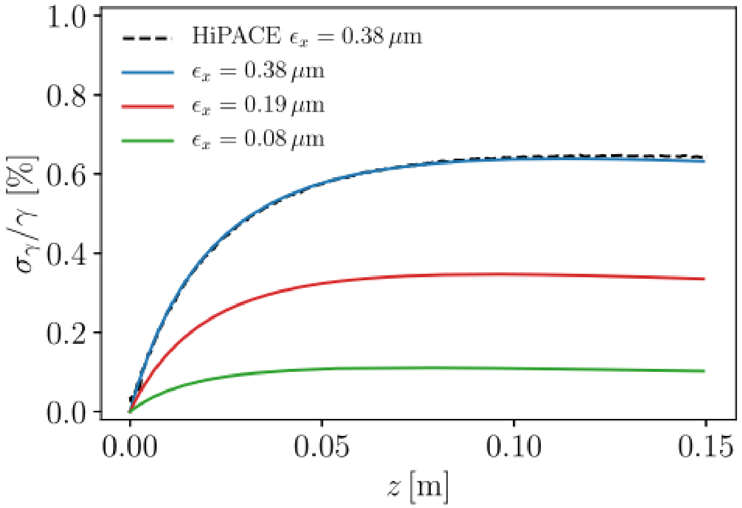
- Uniform E_z along the beam ☺
  The interaction between the electron filament and the bubble boundary makes it possible to flatten E_z for positron beam

- The ability to simultaneously achieve high efficiency and high quality acceleration 😐
  The longitudinal variation of the electron filament increases the slice energy spread of the positron beam, and makes it hard to achieve high efficiency and high beam quality at the same time

How to reach better positron beam quality?

- Using a positron beam with very small beam emittance and narrow size, under 1% level energy spread can be obtained for a small energy transfer efficiency.





$k_p\zeta_{\mathrm{head}}$	Q_w [pC]	E_w^+ [GeV/m]	η [%]
−10.4	54	25.0	2.7
−10.6	57	27.4	3.1
−10.8	52	29.8	3.0
−11.0	36	31.4	2.2

PRAB 23, 121301 (2020)

How to reach better beam quality and higher efficiency?

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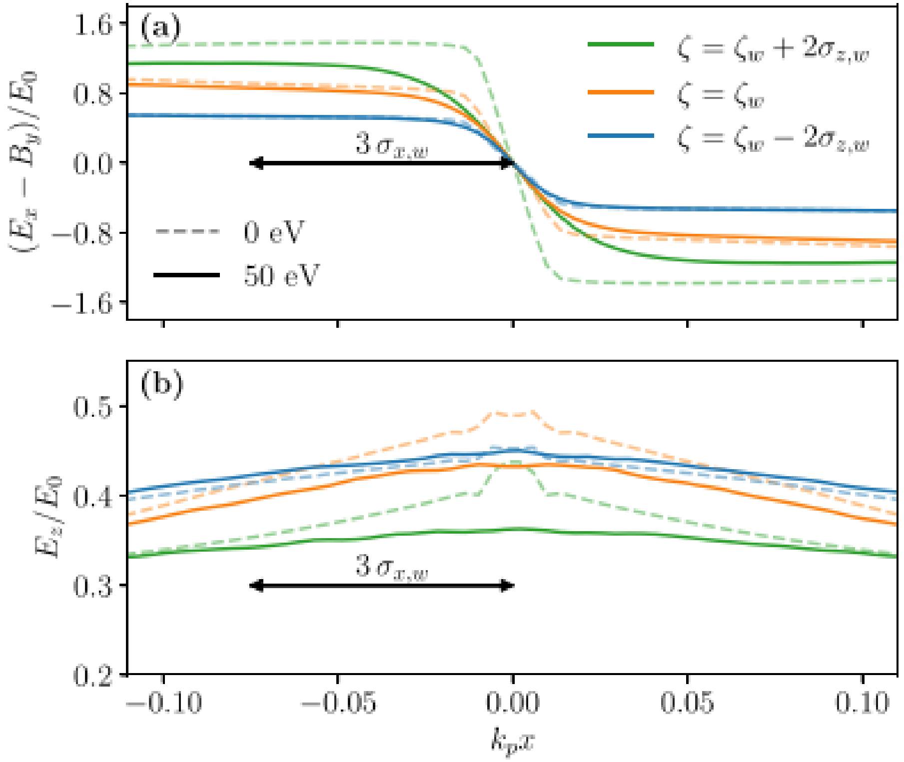
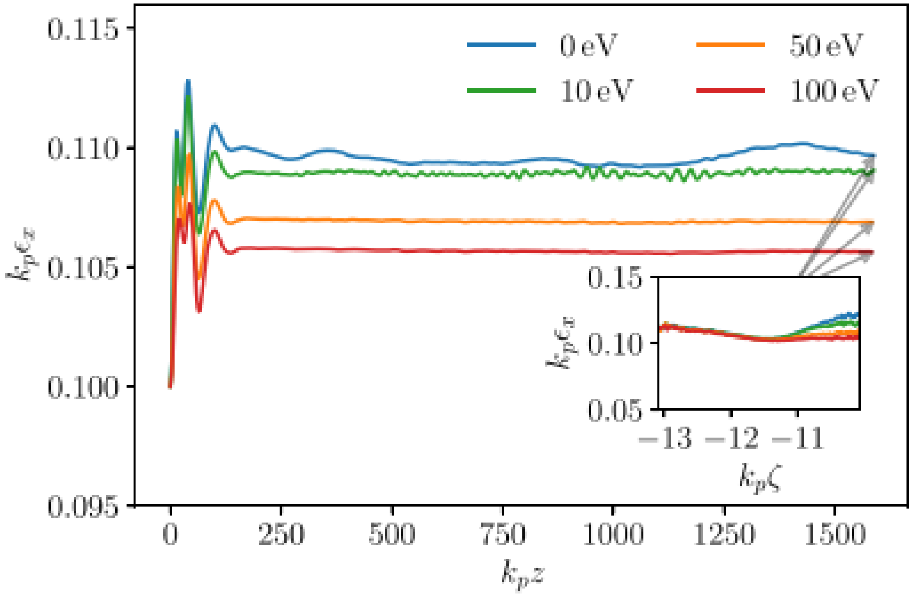
- Use a warm plasma to smooth the variation of the electron filament

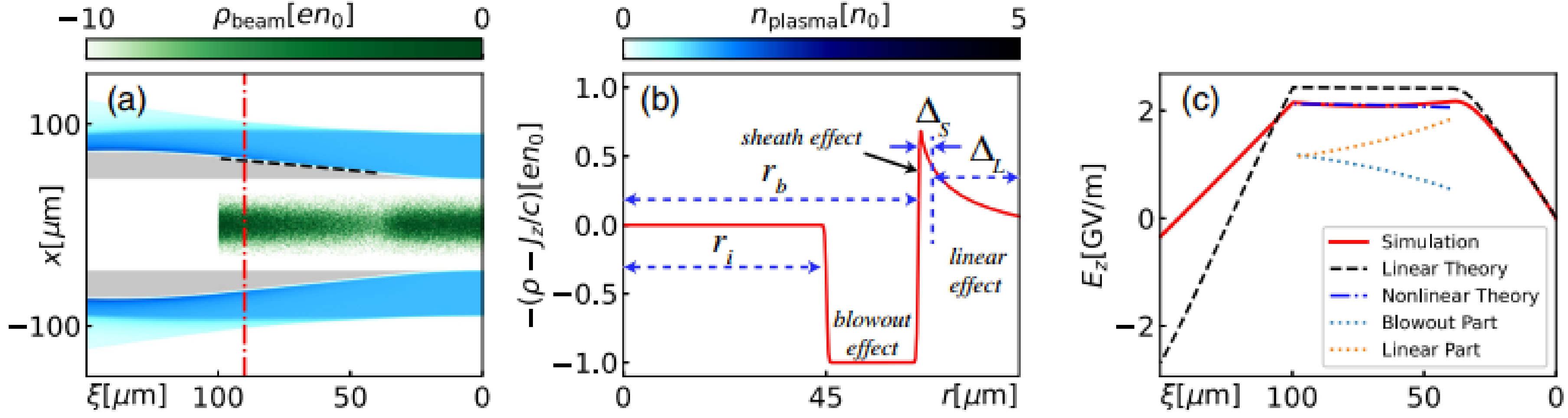




Phys. Plasmas 30, 073104 (2023)

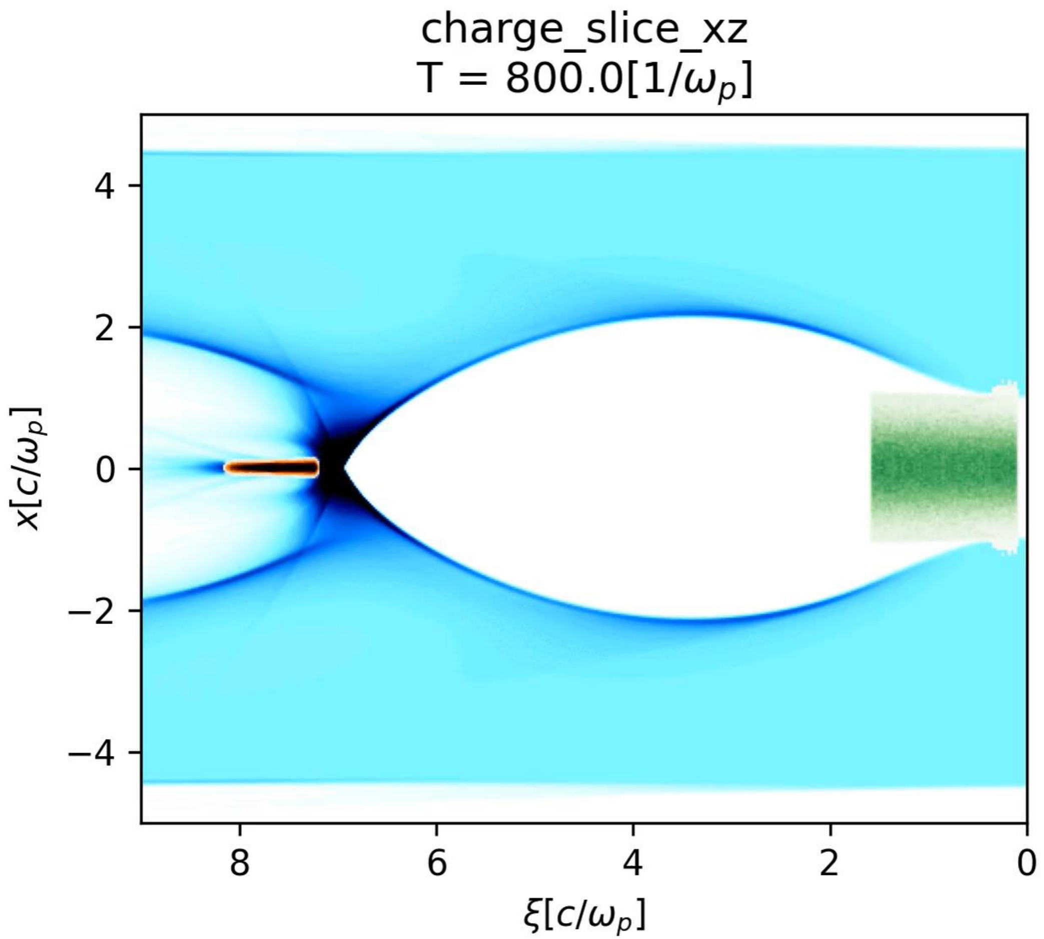
How to reach better beam quality and higher efficiency?

- Use the hollow plasma channel
  - Less electrons are required to focus the positron beam
  - The linear response of the plasma also contributes a lot to the beam loading effect of E_z



PRAB 25, 091303 (2022)

How to reach better beam quality and higher efficiency?



$Q_e = 8\text{nC},\ E_e = 30\text{GeV}$

$Q_p = 1.1\text{nC},\ E_p = 3\text{GeV}, \epsilon_n = 20\text{mm} \cdot \text{mrad}$

After 26m propagation,

$Q_p = 1.1\text{nC},\ E_p = 30.3\text{GeV}, \epsilon_n = 25\text{mm} \cdot \text{mrad}$

$\delta = 0.6\%$

Summary

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- **The blowout regime is able to provide high-efficiency and high quality positron acceleration.**
- **The on-axis electron filament is the key point.**
- **Positron acceleration in the hollow plasma channel is very promising for high-efficiency and low energy spread.**
- **The temperature effects and other effects need further study.**

Thanks for your attention!